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Influence of suspension concentration on cast formation time in pressure filtration

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Abstract

Expressions for the formation rate of an incompressible cast in batchwise pressure filtration are formally derived from Darcy's law and Kynch-theory for pure filtration and filtration with sedimentation and compared with experimental results from the filtration of submicron α-alumina dispersed in water. The influence of suspension concentration on the cast formation time is investigated numerically. A maximum cast formation time is found at a certain suspension concentration ϕ_s for pure filtration and a constant filling height. The value of the maximum cast formation time depends solely on the cast concentration if the filter resistance is negligible. For a constant final cast thickness, the cast formation time always decreases with increasing ϕ_s. The cast formation time in a filtration set-up decreases when particle sedimentation is important, especially when sedimentation and filtration have the same direction. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Pressure filtration; Slip casting; Suspensions

1. Introduction

This paper considers the cast formation time during batchwise pressure filtration of a dispersed suspension of a single particle type. A proper understanding of the influence of the relevant process parameters on the cast formation time is important because this helps to reduce the cast formation time. A reduced cast formation time is advantageous from a manufacturing point of view, because it may result in increased production capacity. Furthermore, a reduced cast formation time may decrease segregation effects which could destroy the resulting cast structure.[1,2]

In batchwise pressure filtration, a suspension is separated into a green cast and a pure filtrate liquid. The green cast is formed at a porous filter which is impermeable for the particles but permeable for the liquid. The driving force is a static pressure difference either accomplished by applying vacuum on the backside of the filter (vacuum casting) or by applying a high pressure on top of the suspension (pressure casting). In pressure casting, either a pressurized gas phase or a piston is used.

The resistance to liquid flow in the filter is constant in pressure filtration; this in contrary to the process of slip casting in which a liquid front penetrates a porous filter (often plaster of Paris) due to capillary forces.[3] Slip casting is not covered in this paper.

We focus on a suspension of dispersed particles because these suspensions are increasingly used in the consolidation of ceramics to obtain superior microstructures.[1,4] For such a suspension, the cast density (cast concentration, packing factor) is high and independent of pressure. Densification with time is also minimal.[1,2] Therefore, cast formation from a suspension of dispersed particles is much simpler to describe than for a flocculated suspension for which complicated mathematical procedures must be used to describe cast formation.[3,5–8] Expressions are derived in this paper that describe the cast formation time of a single batchwise filtration experiment. The larger scope necessary to optimize an industrial filtration operation — including the stage of pressure buildup and loading/unloading — is not followed here.[9,10]

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Three issues are covered in this paper:

- *Theoretical background.* Expressions for cast growth of an incompressible cast because of filtration with and without additional sedimentation are derived formally from Kynch-theory, Darcy's law and appropriate mass balances.
- *Experimental.* Filtration experiments, performed with several suspension concentrations of a submicron α-alumina suspension stabilized by electrostatic forces, are presented to validate the filtration expressions.
- *Simulation results.* The filtration expressions are rewritten to describe the cast formation time for filtration with and without sedimentation. Results are presented for downward filtration (in which filtration and sedimentation have the same direction) and for upward filtration (filtration and sedimentation have opposite directions).

2. Theoretical background

2.1. Pure filtration

For an incompressible cast and a Newtonian liquid, cast formation during batch-wise pressure filtration (without sedimentation, see Fig. 1) is determined by the cast permeability L_c, cast thickness δ_c, the filter resistance R_f, the applied pressure difference ΔP, the cast and suspension (particle volume) concentration ϕ_c and ϕ_s, and the liquid (or filtrate) viscosity η. The concentrations ϕ_s and ϕ_c are defined as the volume of particles in a certain phase divided by the total volume of that phase. The cast concentration is also described by $\phi_c = 1 - \varepsilon$ with ε the porosity of the cast. In the subsequent derivation, all of the above parameters are considered constant during an experiment except for δ_c which increases with time from zero to the maximum value. The maximum value $\delta_{c,\infty}$ follows from an overall balance for the particles, including the initial or filling height of the suspension H_0. For a plan-parallel geometry, the result is:

$$\delta_{c,\infty} = H_0 \frac{\phi_s}{\phi_c} \tag{1}$$

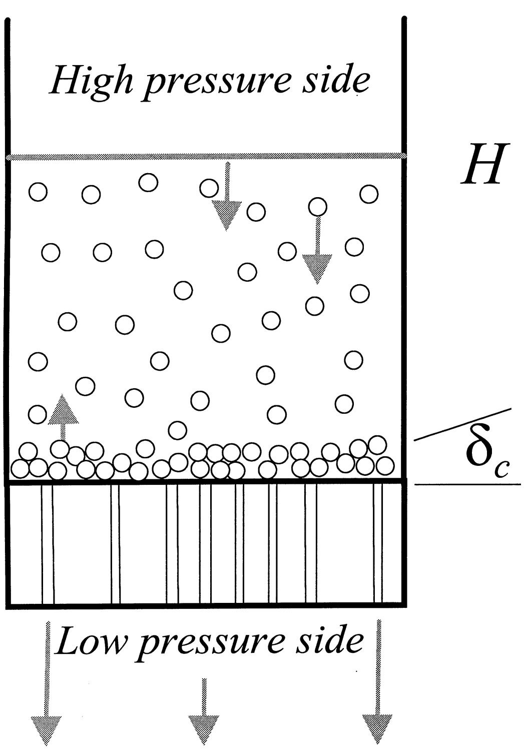

Fig. 1. Pressure filtration set-up (case *F*). The coordinate system originates at the top of the filter and points upward.

The movement of the top of the cast (the cast-suspension discontinuity) is described by Kynch-theory.[11–13] This theory predicts that either kinematic waves or a kinematic shock separate cast from the suspension. From Kynch-theory it can be shown that in a pure filtration process a kinematic shock always occurs because the particle velocity in the suspension ν_s does not depend on the suspension concentration ϕ_s. In this case, a mass balance over the moving cast front is given by:

$$(\nu_s - \nu_c)\phi_s = \left(\nu_{p,c} - \nu_c\right)\phi_c \tag{2}$$

Here, ν_c is the velocity of the cast front and $\nu_{p,c}$ the velocity of particles in the cast. Since we assume that the cast is non-compressible, $\nu_{p,c}$ is set to zero, which results in:

$$\frac{\mathrm{d}\delta_c}{\mathrm{d}t} = \nu_c = -\nu_s \frac{\phi_s}{\phi_c - \phi_s} \tag{3}$$

The velocity of particles in suspension ν_s is given by an overall balance for any horizontal plane through the suspension:

$$\nu_s\phi_s + \nu_L(1 - \phi_s) = J \tag{4}$$

Here, J is the superficial liquid flux which is constant in the entire system (suspension, cast, filter) for a plan-parallel geometry; ν_L is the velocity of the liquid in the suspension. For pure filtration, $\nu_s = \nu_L$ and the result is:

$$\nu_s = J \tag{5}$$

For a Newtonian liquid, the flux J is described by Darcy's law.[14] For uni-directional flow the result is:

$$J = -\frac{L_i}{\eta}\frac{\mathrm{d}P}{\mathrm{d}x} \tag{6}$$

Here, the gravity term $\rho_L \cdot g$ is neglected because in our experiments it is typically a factor 10^3 smaller than the pressure gradient dP/dx. Solution of the equation of continuity for the liquid phase for a planparallel geometry results in J independent of time and place for an incompressible liquid (ρ_L constant) and cast (ϕ_c constant). Integration over cast (c) and filter (f) then results in:

$$J = -\left(R_f + \frac{\delta_c}{L_c}\right)^{-1} \frac{\Delta P}{\eta} \tag{7}$$

Note that we define the pressure difference ΔP as a positive number. Because of the direction of the coordinate system chosen, J is always negative.

Implementation of Eqs. (5) and (7) in (3), and integration with initial condition $\delta_c|_{t=0} = 0$ gives the cast thickness as a function of time:

$$\delta_c = L_c \left\{ \sqrt{R_f^2 + \frac{2\phi_s \Delta P t}{L_c(\phi_c - \phi_s)\eta}} - R_f \right\} \tag{8}$$

This equation can be derived from Bockstal et al.[15] and Bothe et al.[16] using the correlations of Table 2. For a negligible filter resistance, Eq. (8) becomes:[17–19]

$$\delta_c = \sqrt{\frac{2\phi_s L_c \Delta P t}{(\phi_c - \phi_s)\eta}} \tag{9}$$

The height of the upper surface of the suspension H follows from the combination of Eq. (8) with an overall mass balance for the solid particles:

$$\phi_s H_0 = \phi_s (H - \delta_c) + \phi_c \delta_c \tag{10}$$

The result is:

$$H = H_0 - L_c \frac{(\phi_c - \phi_s)}{\phi_s} \left\{ \sqrt{R_f^2 + \frac{2\phi_s \Delta P t}{L_c(\phi_c - \phi_s)\eta}} - R_f \right\} \tag{11}$$

The filtrate volume V_f follows from an overall mass balance for the liquid phase:

$$V_f = A(H_0 - H) \tag{12}$$

Here, A is the surface area perpendicular to flow, which is constant in a pressure filtration experiment with a planar geometry. In (12) it is assumed that the filter has a negligible liquid holdup or is wetted before the suspension is poured on top of the filter.

2.2. Downward filtration with sedimentation

If, during filtration, particles sediment because of gravity, additional phases evolve besides the (growing) cast on the filter and the (diminishing) suspension phase (see Fig. 2). First we consider downward filtration ($F+S$). Here, filtration and sedimentation work in the same direction and a supernatant phase evolves on top of the suspension.[15,16,19,20]

For downward filtration, the particle velocity in the suspension follows from the relative (slip) velocity of particles with the liquid $v_s - v_L$:

$$v_s - v_L = \frac{2(\rho_p - \rho_L) r_p^2}{9\eta} g (1 - \phi_s)^{n-1} \tag{13}$$

This expression is based on the Stokes velocity for a single sphere falling through an infinite continuum together with the Richardson–Zaki[21] equation which accounts for particle hindrance at high suspension concentrations ϕ_s. Here, ρ_p is the density of the particles, ρ_L the density of the liquid, r_p the radius of the particles and g the acceleration due to gravity ($= -9.81$ m/s^2). The power n depends on the Re-number and thus on velocity and particle size; for low Re-numbers ($Re < 0.2$), n can be set at $n = 4.65$.[22] For the base case (Table 1), $Re = 6.5 \times 10^{-7}$ for an infinitely diluted suspension ($\phi_s = 0$). Therefore, $n = 4.65$ can be used in this work. Eq. (5) now becomes:

$$v_s = J + \frac{2(\rho_p - \rho_L) r_p^2}{9\eta} g (1 - \phi_s)^n = J + v_0 \tag{14}$$

Note that v_s, J and v_0 have negative values. Eqs. (7) and (14) are implemented in (3) which gives after integration the following non-explicit expression for the cast thickness δ_c:

$$\delta_c + X \ln\left(1 + \frac{\delta_c}{L_c R_f - X}\right) = -\frac{\phi_s v_0}{\phi_c - \phi_s} t, \quad X = \frac{\Delta P L_c}{\eta v_0} \tag{15}$$

The filtrate volume V_f is again given by Eq. (12). The heights H and H_s (see Fig. 2) follow from Eq. (15) and a simple mass balance over the solid phase which results in:

$$H_s = H_0 - \delta_c \frac{\phi_c - \phi_s}{\phi_s}; \; H = H_s - v_0 t \tag{16}$$

Bockstal et al.[15] were the first to describe combined filtration and sedimentation in downward filtration based on the Ruth-equation and mass-based concentrations. Tiller et al.[20] derive the same expression on a volume-basis. Bothe et al.[16] use the Bockstal-expression but do not define the 'concentration-parameter' κ (see Table 2).

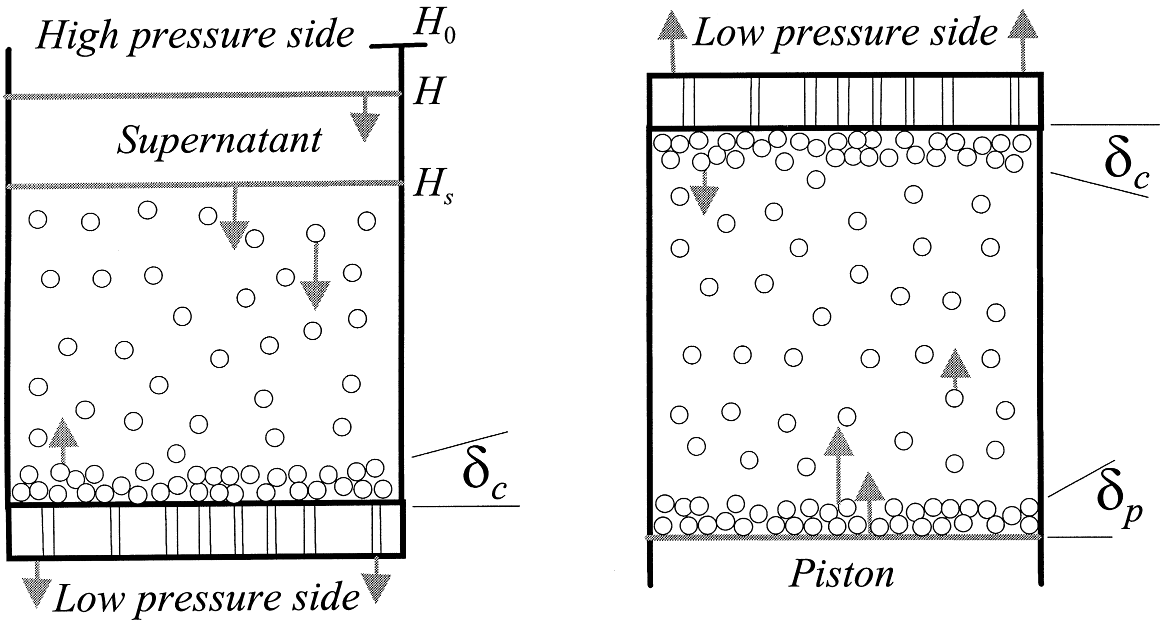

Fig. 2. Filtration combined with sedimentation. Left: downward filtration with sedimentation in the same direction as filtration ($F+S$). Initially the filter was filled to H_0. A supernatant phase evolves at the top of the suspension from H to H_s. The coordinate system originates at the top of the filter and points upward. Right: upward filtration in an inverted press with sedimentation and filtration in opposite directions ($F-S$). A cast is not only formed at the filter (δ_c) but at the piston (δ_p) as well. The coordinate system originates at the bottom of the filter (suspension side) and points downward.

Table 1
Experimental data and base case for simulations

Liquid viscosity η	8.9×10^{-4} Pa s
Filter resistance R_f	4.15×10^{10} m^{-1}
Cast concentration ϕ_c	0.59
Pressure difference ΔP	9.85×10^{4} Pa
Cast permeability L_c	9.9×10^{-17} m^2
Initial filling height H_0	0.152 m
Particle density ρ_p	3940 kg/m^3
Liquid density ρ_L	1000 kg/m^3
Particle radius r_p	300 nm
Hindrance power n	4.65
Gravity acceleration g	±9.81 m/s^2

Expressed in the above defined parameters, the expression by these authors is as follows:

$$t=-\nu_0^{-1}\left(\frac{L_c(\phi_c-\phi_s)}{\phi_s}\left(R_f-\frac{\Delta P}{\eta\nu_0}\right)\left(\exp\left(-\frac{\eta\nu_0\phi_s(H_0-H)}{\Delta PL_c(\phi_c-\phi_s)}\right)-1\right)-(H_0-H)\right) \tag{17}$$

The conversions made are summarized in Table 2. From a mathematical point of view, it was surprising to us that the outcome of Eq. (17) was identical to the outcome of (15) and (16) though the equations look so different. For a negligible filter resistance, Philipse et al.[19] also describe combined filtration and sedimentation and arrive at (17) for $R_f=0$.

2.3. Upward filtration with sedimentation

Particles can also be filtered in an upward direction, either because of a vacuum applied on the backside of the filter[23] or because of a piston placed below the suspension.[6,8] This latter set-up is shown in Fig. 2 and will be discussed in more detail in this section. In an inverted press, the filter is located above the slurry and a piston below the slurry. The piston presses the liquid upward through cast and filter. If the cast permeability and the liquid flux are high enough, a cast layer develops at the filter (δ_c). Because of sedimentation a cast will always grow at the piston (δ_p). Cast formation on the filter is again described by (15) but note that the coordinate system now points downward (see Fig. 2) which implies that g and ν_0 now have positive values. Particle velocity in the suspension ν_s can be directed toward or away from the filter.[23] The latter situation becomes more realistic in the course of the process because a cast is built up at the filter and the filtration flux J decreases. If particles move away from the filter, cast is no longer formed at the filter and particles only sediment toward the piston. In that case, a clear supernatant layer will develop between the cast on the filter (δ_c) and the suspension. The critical cast thickness δ_c above which particles will move away from the filter can be calculated from (15):

$$\delta_{c,\mathrm{critical}}=X-L_cR_f \tag{18}$$

Table 2
Different definitions in downward filtration with sedimentation

Bockstal et al., 1985[15]	Philipse et al., 1990[19]	Tiller et al., 1995[20]	Bothe et al., 1997[16]	This paper
	k_c		α^{-1}	L_c
			κ	$(\phi_c-\phi_s)\phi_s^{-1}$
V/A	$h(t)$	v	v	H_0-H
		R_m	β	R_f
$f.A$		Gu_{sR}	s	$-\eta v_0\phi_s/(\Delta PL_c(\phi_c-\phi_s))$
$a\ A^2$			$2q$	$\eta\phi_s/(\Delta PL_c(\phi_c-\phi_s))$
bA			r	$\eta R_f/\Delta P$
v_s	v_0	u_{sR}	W_{sed}	$-v_0$
c^a				$\rho_s\phi_s(1-\phi_s)^{-1}$
α				$(1-\phi_s)/(L_c\rho_s(\phi_c-\phi_s))$
		c		$\phi_s\phi_c(\phi_c-\phi_s)^{-1}$
		α_{av}		$(L_c\phi_c)^{-1}$
		ω_c		$\delta_c\phi_c$
μ	η	μ	η	η
	U	q		$-J$
	$l(t)$	L	h_K	δ_c
	j			$\phi_s/(\phi_c-\phi_s)$
	$1-\varepsilon_s$			ϕ_s
	$1-\varepsilon_c$	ε_{sav}		ϕ_c
	$-\Delta P$	p	ΔP	ΔP

[a] "The slurry concentration in kg of solids per unit volume of filtrate."

For $\delta_c > \delta_{c,\text{critical}}$, (15) will not give a solution which is in accordance with the physical picture, namely that the suspension moves away from the filter leaving a supernatant phase behind. Cast formation on the piston (velocity cast front v_p, thickness δ_p) is given by a mass balance over the moving boundary δ_p:

$$\phi_c(v_{\text{piston}}-v_p)=\phi_s(v_s-v_p) \tag{19}$$

Because v_{piston} equals the liquid flux J, (19) rewrites to:

$$\frac{d\delta_p}{dt}=|v_p-v_{\text{piston}}|=\frac{\phi_s(v_s-J)}{\phi_s-\phi_c}=\frac{\phi_s}{\phi_s-\phi_c}v_0 \rightarrow \quad \delta_p=\frac{\phi_s}{\phi_s-\phi_c}v_0 t \tag{20}$$

With (15) and (20) pressure filtration in an inverted press is described.

3. Experimental

In this section, expressions (11) and (16) are compared with pressure filtration experiments with a suspension of submicron α-alumina, which is a typical example of cast formation in ceramic engineering.

3.1. Set-up

A simple pressure filtration set-up was constructed from a glass tube (ID 20 mm, height 200 mm) with a glass fritted plate (pore size~5–10 μm, thickness 2 mm) melted in the tube at right angles. On top of the fritted plate, a polymer membrane was placed (pore size ~800 nm, thickness ~0.13 mm, type ME-27, Schleicher and Schuell, Dassel, Germany). The filter (fritted glass plate plus polymer membrane) resistance R_f was measured using pure water as permeant. With Eq. (7) and $\delta_c=0$, the filter resistance could be determined as $R_f=4.15\times10^{10}\,\text{m}^{-1}$.

Suspensions were prepared by mixing AKP-15 powder (Sumitomo, Tokyo, Japan) with 0.02 M nitric acid in pure (distilled) water. The suspensions were ultrasonically vibrated (Model 250 Sonifier, Branson Ultrasonics, Danbury, CT) to break up the agglomerates and filtered over a 200 μm filter to remove remaining aggregates. The mean particle radius r_p (=300 nm) was determined by an optical particle size technique (Microtrac X-100, Leeds and Northrup, North Wales, PA, USA). The filtration set-up was filled with suspension up to 152 mm and vacuum (~15 mbar) was applied using a water jet pump, while the pressure on the low pressure side was measured electronically. The location of the top of the suspension (H and H_s) was measured with an accuracy ~0.5 mm.

3.2. Experimental results

Results of the experiments are shown in Fig. 3. For the highest suspension concentration, no supernatant layer could be observed, while a supernatant layer developed in time with a maximum thickness of 3.5 mm for $\phi_s=0.0162$ and 5 mm for $\phi_s=0.0051$. The cast concentration ϕ_c (=0.59) was determined from the final cast thickness $\delta_{c,\infty}$. Measurements were best described [using Eq. (11) for filtration without sedimentation] by a value for the cast permeability L_c of 9.9×10^{-17} m^2. This value for L_c is roughly 4 times lower than that predicted by the well-known Carman–Kozeny equation for single-sized spherical particles:[24]

$$L_{CK} = \frac{(1-\phi_c)^3 r_p^2}{45\phi_c^2} \tag{21}$$

It is also roughly 4 times lower than the permeability of an AKP-15 compact sintered at 1100°C.[25] For $t < 9000$ s, the deviation between model and measurement is within the measurement error but the deviation is ~5 mm for the measurements at t~9000 s.

For $\phi_s = 0.0162$, the predictions for the case of downward filtration with sedimentation is plotted as well [Eq. (16)]. Now measurements of H_s are better described, but for H the deviation between experiment and model increases. We have yet no explanation for this result.

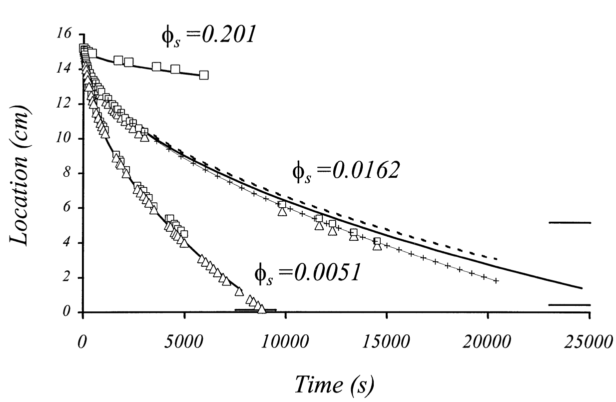

Fig. 3. Measured suspension height H (squares) and H_s (triangles) for different suspension concentrations ϕ_s. Modelled height H for case of no sedimentation [Eq. (11), solid lines] and for downward filtration combined with sedimentation [Eq. (16), H is the dashed line and H_s the crossed line]. Horizontal lines show the final cast thickness $\delta_{c,\infty}$ for the three concentrations. According to Eq. (22), the final thickness is only reached after 53 h for the highest concentration ($\phi_s = 0.201$). Data from Table 1.

4. Simulation results

In this section, the cast formation time is calculated for the case of pure filtration and the case of filtration with sedimentation. For the latter case, also the influence of the direction of sedimentation (in relation to the direction of filtration) is investigated numerically. We focus on the influence of the suspension volume and concentration ϕ_s because these are often the only parameters that can be changed readily for a certain suspension recipe and filtration set-up (i.e. liquid and powder properties cannot be changed easily). To investigate the influence of ϕ_s on the cast formation time t_c, other parameters must be kept constant. In pressure filtration it is in this respect essential to define whether the final cast thickness $\delta_{c,\infty}$ or the initial filling height H_0 is kept constant. We start with the latter case.

4.1. Cast formation time for pure filtration and a constant filling height H_0

Cast formation time t_{c,H_0} for an experiment with constant filling height H_0, is given by implementing (1) in (8) and rewriting to an explicit equation for t_{c,H_0}:

$$t_{c,H_0} = \frac{\eta(\phi_c-\phi_s)}{2\Delta P}\left\{\frac{H_0^2\phi_s}{L_c\phi_c^2} + \frac{2H_0R_f}{\phi_c}\right\} \tag{22}$$

For the base case given in Table 1, t_{c,H_0} is evaluated for $0 < \phi_s < 0.59$ in Fig. 4. The limit of $t_{,c,H_0}$ when ϕ_s approaches zero, is given by:

$$\lim_{\phi_s\to0} t_{c,H_0} = \frac{\eta H_0 R_f}{\Delta P} \tag{23}$$

This is the time necessary to drain a pure liquid through the filter. As can be observed, cast formation time increases with increasing H_0 for all values of the suspension concentration ϕ_s. Cast formation time only increases with ϕ_s for values of ϕ_s on the LHS of $\phi_{s,\text{opt}}$. The value of $\phi_{s,\text{opt}}$ can be derived from differentiating Eq. (22) once with respect to ϕ_s, which gives the following expression:

$$\phi_{s,\text{opt}} = \phi_s\Big|_{\frac{dt_{c,H_0}}{d\phi_s}=0} = \frac{\phi_c}{2}\left(1-\frac{2L_cR_f}{H_0}\right) \tag{24}$$

The limit for an infinitely low filter resistance R_f is given by:

$$\phi_{s,\text{opt,lim}} = \frac{\phi_c}{2} \tag{25}$$

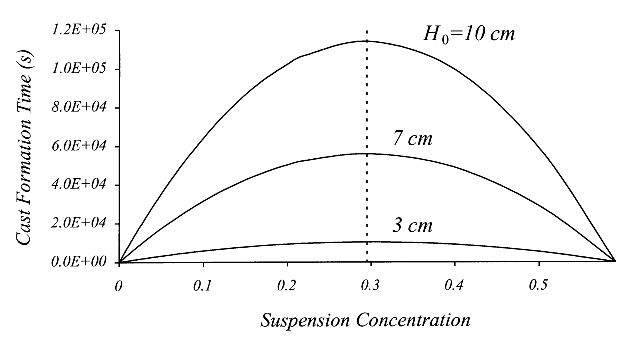

Fig. 4. Cast formation time t_c for pure filtration and an initial suspension height H_0. Time t_{c,H_0} increases with ϕ_s only on the LHS of the dashed line, given by Eq. (25). Data from Table 1.

For our base case, Eq. (25) can be used instead of (24) and the result of (25) is plotted in Fig. 4. To decrease cast formation time, ϕ_s should be decreased for $\phi_s < \phi_{s,\mathrm{opt,lim}}$ but increased for $\phi_s > \phi_{s,\mathrm{opt,lim}}$.

4.2. Cast formation time for pure filtration and a constant final cast thickness $\delta_{c,\infty}$

For a constant final cast thickness $\delta_{c,\infty}$ Eq. (8) can be rewritten directly to give an expression for $t_{c,\delta_{c,\infty}}$. For $\phi_s \to 0$, $t_{c,\delta_{c,\infty}}$ goes to infinity as can be seen in Fig. 5. No maximum of $t_{c,\delta_{c,\infty}}$ can be observed and $t_{c,\delta_{c,\infty}}$ decreases monotonically with increasing ϕ_s. Cast formation time increases for all values of ϕ_s on increasing $\delta_{c,\infty}$.

One is often more interested in the cast thickness after densification δ_{dense} ($=\phi_c\delta_{c,\infty}$, assuming $\phi_{c,\mathrm{dense}}=1$) instead of $\delta_{c,\infty}$. For a negligible cast resistance, we arrive at $t_{c,\delta_{\mathrm{dense}}}$ using Eqs. (9) and (21):

$$t_{c,\delta_{\mathrm{dense}}} = \frac{45(\phi_c-\phi_s)\eta\delta_{\mathrm{dense}}^2}{2\phi_s(1-\phi_c)^3 r_p^2 \Delta P} \tag{26}$$

Clearly, $t_{c,\delta_{\mathrm{dense}}}$ can be decreased by decreasing ϕ_c,η (e.g. by increasing temperature) and δ_{dense} or by increasing ϕ_s (higher solids loading), particle radius r_p or the pressure difference ΔP. The packing factor ϕ_c can be decreased by the use of non-spherical particles or interparticle attraction in suspension resulting in flocculation. The packing factor ϕ_c of sedimented flocs is often much lower than of dispersed particles, but increases rapidly due to densification of the cast. Note that this lower ϕ_c is the reason why flocculation decreases cast formation time in pressure filtration and not its influence on the viscosity of the suspension, as is often reported in literature. As shown clearly in Eq. (26), it is the viscosity of the pure liquid η (without particles) that determines cast formation time and not the viscosity of the liquid-particle mixture. Note further that flocculation will decrease the cast formation time further because the sedimentation velocity of flocs is generally much higher than for dispersed particles because of the larger effective size. Flocculation and highly non-spherical particles are not always appropriate for cast formation because the resulting cast structure may be inhomogeneous resulting in anomalous drying and sintering.

4.3. Cast formation time for combined filtration and sedimentation

For downward filtration with sedimentation in the same direction ($F+S$), cast formation time follows directly from (15):

$$t_c = -\frac{\phi_c-\phi_s}{\phi_s v_0}\left(\delta_{c,\infty} + X\ln\left(1+\frac{\delta_{c,\infty}}{L_cR_f-X}\right)\right), \quad X=\frac{\Delta P L_c}{\eta v_0} \tag{27}$$

For a very low sedimentation velocity v_0 (i.e. negligible sedimentation), the outcome of (27) is equal to the outcome of (8). For a very high v_0 (only sedimentation, no filtration), the result of (27) is the same as if only sedimentation would occur, namely:

$$t_c|_{\text{solely sedimentation}} = -\frac{\phi_c-\phi_s}{\phi_s v_0}\delta_{c,\infty}.$$

For upward filtration with sedimentation in the opposite direction in an inverted press ($F-S$), cast formation time is defined as the moment when all particles have moved into the cast on the filter (δ_c) or into the cast on the piston (δ_p). At that moment, the summation of δ_c and δ_p equals the final cast thickness $\delta_{c,\infty}$. For an

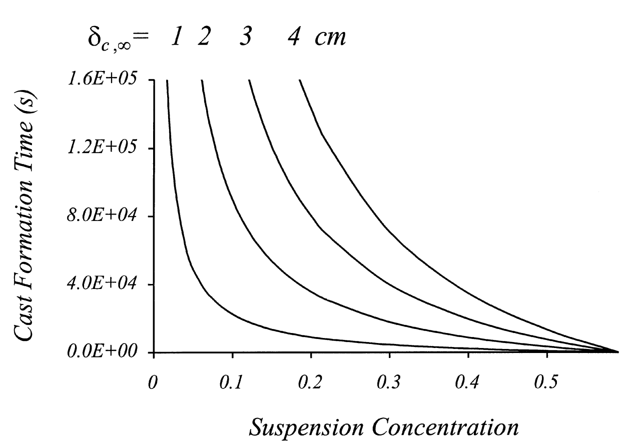

Fig. 5. Cast formation time for pure filtration and a constant final cast thickness $\delta_{c,\infty}$. Initial suspension height H_0 follows from Eq. (1). Data from Table 1.

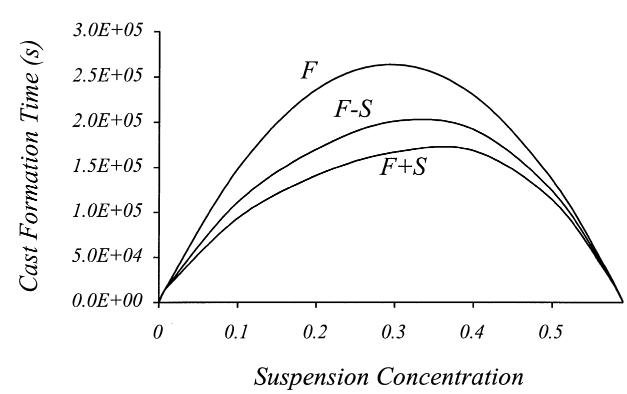

Fig. 6. Cast formation time for pure filtration (no sedimentation, F), for sedimentation in the direction of filtration (downward filtration, $F+S$) and for sedimentation in the opposite direction as filtration (upward filtration, $F-S$). The initial filling height H_0 is taken as constant. Data from Table 1.

inverted press, the cast formation time can only be calculated by trial-and-error using Eqs. (15) and (20) and $\delta_{c,\infty} = \delta_c + \delta_p$.

Fig. 6 depicts model results for the cast formation time for pure filtration (F) and filtration and sedimentation combined for downward filtration ($F+S$) and for upward filtration (inverted press, $F-S$). For the inverted press ($F-S$), the particle velocity never became negative in our model calculations (which would imply that the particles move away from the filter, leaving behind a supernatant layer between cast on the filter and suspension). Therefore (15) could be used in all situations. The simulations show that sedimentation decreases the cast formation time and most effectively for downward filtration with sedimentation ($F+S$). This can be explained by the fact that for case $F+S$ liquid is separated from the suspension at two locations (instead of one location, as for case F and for case $F-S$), namely at the filter as well as at the top of the suspension, where a supernatant layer evolves. Case $F-S$ only gives a higher cast formation rate that case F because part of the cast is formed at the piston, which is advantageous because cast formed at that location does not increase the resistance to filtration.

5. Summary

Equations for cast growth and cast formation time in batchwise pressure filtration with a non-compressible cast were formally derived from Kynch-theory for pure filtration and filtration with sedimentation. The equations were in fair agreement with experiments on a electrostatically stabilized submicron α-alumina powder indicating that the cast does not densify significantly with time for dispersed particles.

For a constant filling height, a maximum in cast formation time t_c is found when the suspension concentration ϕ_s is changed. For a negligible filter resistance, the value of ϕ_s at the maximum t_c depends only on the cast concentration. However, for a constant final cast thickness, the cast formation time t_c decreases monotonically with increasing ϕ_s. Sedimentation always decreases the cast formation time. In a so-called inverted press, in which sedimentation and filtration have opposite directions, cast formation time is intermediate between pure filtration and segregation in the direction of filtration.

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References

1. Velamakanni, B. V. and Lange, F. F., Effect of interparticle potentials and sedimentation on particle packing density of bimodal particle distributions during pressure filtration. *J. Am. Ceram. Soc.*, 1991, **74**, 166–172.
2. Chang, J. C., Velamakanni, B. V., Lange, F. F. and Pearson, D. S., Centrifugal consolidation of Al_2O_3 and Al_2O_3/ZrO_2 composite slurries vs interparticle potentials: particle packing and mass segregation. *J. Am. Ceram. Soc.*, 1991, **74**, 2201–2204.
3. Tiller, F. M. and Tsai, C. D., Theory of filtration of ceramics: I, slip casting. *J. Am. Ceram. Soc.*, 1986, **69**, 882–887.
4. Binner, J. G. P. and Murfin, A. M., The effect of temperature, heating method and state of dispersion on the vacuum filter casting of alumina suspensions. *J. Eur. Ceram. Soc.*, 1998, **18**, 791–798.
5. Landman, K. A. and White, L. R., Predicting filtration time and maximizing throughput in a pressure filter. *AIChE J.*, 1997, **43**, 3147–3159.
6. Kellett, B. J. and Lin, C.-Y., Mechanics of constant-rate filter pressing of highly flocculated slurries. *J. Am. Ceram. Soc.*, 1997, **80**, 381–393.
7. Civan, F., Practical model for compressive cake filtration including fine particle invasion. *AIChE J.*, 1998, **44**, 2388–2398.
8. Lin, C.-Y. and Kellett, B. J., General observations of constant flow rate filter pressing. *J. Am. Ceram. Soc.*, 1998, **81**, 2093–2108.
9. Brown, T. R. Designing batch pressure filters. Chem. Eng., 1982, July, **89**, 58–63.
10. Brown, T. R., Easily optimize batch pressure filtration. Chem. Eng. Progr., 1998, **94**, 45–50.
11. Kynch, G. J., A theory of sedimentation. *Trans. Faraday Soc.*, 1952, **48**, 166–176.
12. Probstein, R. F., *Physicochemical Hydrodynamics, An Introduction*. Butterworths, Boston, 1989.
13. Russel, W. B., Saville, D. A. and Schowalter, W. R., *Colloidal Dispersions*. Cambridge University Press, 1989.
14. Bird, R. B., Stewart, W. E. and Lightfoot, E. N., *Transport Phenomena*. Wiley, New York, 1960.
15. Bockstal, F., Fouarge, L., Hermia, J. and Rahier, G., Constant pressure cake filtration with simultaneous sedimentation. *Filtr. Sep.*, 1985, **22**, 255–257.
16. Bothe, C., Esser, U. and Fechtel, T., Experimental and theoretical studies on filtration with superimposed sedimentation in pressure filters. *Chem. Ing. Techn.*, 1997, **69**, 903–912.
17. Lange, F. F. and Miller, K. T., Pressure filtration: consolidation kinetics and mechanics. *Am. Ceram. Soc. Bull.*, 1987, **66**, 1498–1504.
18. Lange, F. F., Powder processing science and technology for increased reliability. *J. Am. Ceram. Soc.*, 1989, **72**, 3–15.
19. Philipse, A. P., Bonekamp, B. C. and Veringa, H. J., Colloidal filtration and (simultaneous) sedimentation of alumina and silica suspensions: influence of aggregates. *J. Am. Ceram. Soc.*, 1990, **73**, 2720–2727.
20. Tiller, F. M., Hsyung, N. B. and Cong, D. Z., Role of porosity in filtration: XII. Filtration with sedimentation. *AIChE J.*, 1995, **41**, 1153–1164.
21. Richardson, J. F. and Zaki, W. N., Sedimentation and fluidization. *I. Trans. Inst. Chem. Eng.*, 1954, **32**, 35–53.
22. Wallis, G. B., *One-dimensional Two-Phase Flow*. McGraw–Hill, New York, 1969.
23. Rushton, A. and Rushton, A., Sedimentation effects in filtration. *Filtr. Sep.*, 1973, **10**, 267–272.
24. Dullien, F. A. L., *Porous Media, Fluid Transport and Pore Structure*. Academic Press, New York, 1979.
25. Biesheuvel, P. M. and Verweij, H., Design of ceramic membrane supports: permeability, tensile strength and stress. *J. Membrane Sci.*, 1999, **156**, 141–152.